Wavelets and its applications

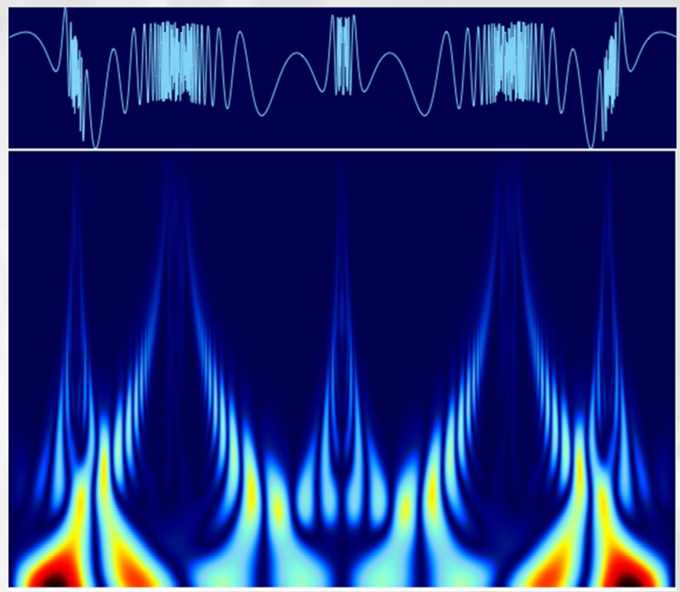
Wavelets are a recently developed signal processing tool enabling the analysis on several timescales of the local properties of complex signals that can present nonstationary zones. They lead to a huge number of applications in various fields, such as, for example, geophysics, astrophysics, telecommunications, imagery and video coding. They are the foundation for new techniques of signal analysis and synthesis and find beautiful applications to general problems such as compression and denoising. The propagation of wavelets in the scientific community, academic as well as industrial, is surprising.

First of all, it is linked to their capacity to constitute a tool adapted to a very broad spectrum of theoretical as well as practical questions. Let us try to make an analogy: the emergence of wavelets could become as important as that of Fourier analysis. A second element has to be noted: wavelets have benefited from an undoubtedly unprecedented trend in the history of applied mathematics.

Indeed, very soon after the grounds of the mathematical theory had been laid in the middle of the 1980s [MEY 90], the fast algorithm and the connection with signal processing [MAL 89] appeared at the same time as Daubechies orthogonal wavelets [DAU 88]. This body of knowledge, diffused through the Internet and relayed by the dynamism of the research community enabled a fast development in numerous applied mathematics domains, but also in vast fields of application.

Thus, in less than 20 years, wavelets have essentially been imposed as a fruitful mathematical theory and a tool for signal and image processing. They now therefore form part of the curriculum of many pure and applied mathematics courses, in universities as well as in engineering schools.

By omitting the purely mathematical contributions and focusing on applications, we may identify three general problems for which wavelets have proven very powerful. The first problem is analysis, for scrutinizing data and sounding out the local signal regularity on a fine scale.



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Wavelets constitute a mathematical “zoom” making it possible to simultaneously describe the properties of a signal on several timescales. The second problem is denoising or estimation of functions. This means recovering the useful signal while we only observe a noisy version thereof. Since the denoising methods are based on representations by wavelets, they create very simple algorithms that, due to their adaptability, are often more powerful and easy to tune than the traditional methods of functional estimation.

The principle consists of calculating the wavelet transform of observations, then astutely modifying the coefficients profiting from their local nature and, finally, inversing the transformation. The third problem is compression and, in particular, compression of images where wavelets constitute a very competitive method. Due to generally very sparse representations, they make it possible to reduce the volume of information to be coded. In order to illustrate this point, we can consider two leading applications whose impact has propagated well beyond the specialists in the field.

The first application relates to the storage of millions of fingerprints by the FBI and the second is linked to the new standard of image compression JPEG 2000, which is based on wavelets.

Let us quote, for example, the numerical solution of partial derivative equations or even, more to the point, the simulation of paths for fractional Brownian processes.

Wavelets have a few interesting applications, some of which are mentioned below. However, the applications of wavelets by themselves are limited. The ideas behind wavelets, which we will be covering in this lecture and future lectures, are more important. The most common use of wavelets is in signal processing applications. For example:

Compression applications. If we can create a suitable representation of a signal, we can discard the least significant" pieces of that representation and thus keep the original signal largely intact. This requires a transformation which separates the important parts of the signal from less important parts.

In the simplest case, we can decompose a signal into two parts: a low frequency part, which is some sort of average of the original signal, and a high frequency part, which is what remains after the low frequency part is subtracted from the original signal.

If we are interested in the low frequency part and hence discard the high frequency part, what remains is a smoother representation of the original signal with its low frequency components intact.

Alternatively, if we are most interested in the high frequency part, we may be able to discard the low frequency part instead. This approach that of decomposing a signal into two parts, is common for all wavelets. Also fundamental to wavelet analysis is a hierarchical decomposition, in which we may apply further transforms to an already decomposed signal.

Edge detection. With this application it is most important to identify the areas in which the input image changes quickly. We can discard the smooth (low frequency) parts. The simplest wavelet basis, the Haar basis (to be discussed later) is suitable for this application. Along this vein, the book by Strang and Nguyen describes a widely used application of wavelets, fingerprint compression, in which edge detection figures prominently.

Graphics. Two prominent uses of wavelets in graphics include

1. Curve and surface representations; and
2. Wavelet radiosity.

References:

Introduction to Wavelets Lecture #1: Tuesday, 30 September 1997 Lecturer: Denis Zorin Scribe: John Owens

Wavelets and their Applications: Edited by Michel Misiti, Yves Misiti, Georges Oppenheim ,Jean-Michel Poggi

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You gain strength, courage, and confidence by every experience in which you really stop to look fear in the face. You must do the thing which you think you cannot do.

Eleanor Roosevelt